1. A right triangle has side lengths of 4 and 8. What is the length of the hypotenuse (the longest side)?
   (a) $4\sqrt{2}$    (b) 10    (c) $4\sqrt{5}$    (d) $4\sqrt{3}$    (e) 12

2. An isosceles triangle has a base length of 6 and a side length 12. What is the area of the triangle?
   (a) $9\sqrt{15}$    (b) 27    (c) $35\sqrt{3}$    (d) $35\sqrt{5}$    (e) 35

3. A cone has a height of 6 and a base circle with a radius of 4. What is the volume of the cone?
   (a) $16\pi$    (b) $32\pi$    (c) $16\pi^2$    (d) $24\pi$    (e) $12\pi$

4. Suppose you are blindly pulling two marbles with replacement from a bag of 14 marbles with 7 red, 4 yellow and 3 black. What is the probability that you pick one red and one black marbles?
   (a) $\frac{2}{7}$    (b) $\frac{3}{28}$    (c) $\frac{2}{11}$    (d) $\frac{3}{4}$    (e) $\frac{1}{2}$

5. The boiling temperature of water is at 100 degrees Celsius. What is the boiling temperature in Fahrenheit? Note that the Fahrenheit($F^\circ$) to Celsius($C^\circ$) conversion formula is $C^\circ = \frac{5}{9}(F^\circ - 32^\circ)$.
   (a) 200    (b) 50    (c) 212    (d) 32    (e) 235

6. A line intersects two points $(-1, -1)$ and $(1, 3)$. Which of the following expressions represents the line?
   (a) $y = x$    (b) $y = x + 1$    (c) $y = x - 1$    (d) $y = 2x$    (e) $y = 2x + 1$

7. Solve for $t$ in $23 + 18t = 6t - 37$.
   (a) $t = 3$    (b) $t = 5$    (c) $t = -3$    (d) $t = -5$    (e) $t = -2$

8. Solve for $r$ in $\frac{r}{3} + 22 = 37$.
   (a) $r = 15$    (b) $r = 3$    (c) $r = 30$    (d) $r = 10$    (e) $r = 45$

9. Calculate $15 - 3 \times 5$.
   (a) 60    (b) 30    (c) 0    (d) 40    (e) 50
10. Amy plans to plant flowers and buys 23.5 pounds of potting soil. What is the minimum number of pots that Amy needs if each pot is filled with 1.35 pounds, and she uses all the potting soil?
(a) 17  (b) 15  (c) 18  (d) 14  (e) 19

11. Angie, Bob, Carlos, and Dan are randomly seated around a square table with one person to a side. What is the probability that Angie and Carlos are seated opposite each other?
(a) $\frac{1}{4}$  (b) $\frac{1}{3}$  (c) $\frac{1}{2}$  (d) $\frac{2}{3}$  (e) $\frac{3}{4}$

12. Two congruent squares, $ABCD$ and $PQRS$, have side length 15. They overlap to form the 15 by 25 rectangle $AQRD$ as shown. What percentage of the area of rectangle $AQRD$ is shaded? (Figure not drawn to scale!)

```
A    P    B    Q
D     S    C    R
```

(a) 15  (b) 18  (c) 20  (d) 24  (e) 25

13. There are 180 students at Coldspring Middle School, where the ratio of boys to girls is 5:4. There are 270 students at Warmspring Middle School, where the ratio of boys to girls is 4:5. The two schools hold a dance party and all the students from both schools attend. What fraction of the students at the dance party are girls?
(a) $\frac{7}{18}$  (b) $\frac{7}{15}$  (c) $\frac{22}{45}$  (d) $\frac{1}{2}$  (e) $\frac{23}{45}$

14. Let $w$, $x$, $y$, and $z$ be whole numbers. If $2^w \times 3^x \times 5^y \times 7^z = 588$, what does $2w + 3x + 5y + 7z$ equal?
(a) 21  (b) 25  (c) 27  (d) 35  (e) 56

15. How many rectangles are in the figure below?

```
  +---+---+
|   |   |
+---+---+
  +---+---+
|   |   |
+---+---+
  +---+---+
|   |   |
+---+---+
```

(a) 8  (b) 9  (c) 10  (d) 11  (e) 12
16. If \( \frac{a}{c} | \frac{b}{d} = ad - bc \), then what is \( \frac{3}{4} \)?
   (a) \(-2\) (b) \(-1\) (c) \(0\) (d) \(1\) (e) \(2\)

17. The graph shows the price of five gallons of gasoline during the first ten months of the year. By what percent is the highest price higher than the lowest price?
   (a) 50% (b) 62% (c) 70% (d) 89% (e) 100%

18. A circle with radius 1 is inscribed in a square and circumscribed about another square as shown. Which of the following is closest to the ratio of the shaded area in the circle to the total shaded area?
   (a) \(\frac{1}{2}\) (b) \(1\) (c) \(\frac{3}{2}\) (d) \(2\) (e) \(\frac{5}{2}\)

19. Ken takes a long bike ride on a hilly highway. The graph indicates the miles traveled during the time of his ride. What is Ken’s average speed for the entire ride in miles per hour?
   (a) 2 (b) 2.5 (c) 4 (d) 4.5 (e) 5
20. What is the sum of the mean, median, and mode of the numbers 2, 3, 0, 3, 1, 4, 0, 3?
   (a) 6.5   (b) 7   (c) 7.5   (d) 8.5   (e) 9

21. As Amy rides her bicycle on a long straight road, she spots Bob skating in the same direction half a mile in front of her. After she passes him, she can see him in her rear view mirror until he is half a mile behind her. Amy rides at a constant rate of 12 miles per hour, and Bob skates at a constant rate of 8 miles per hour. How many minutes passed between when Amy first spotted Bob and when she lost him in her rear view mirror?
   (a) 6   (b) 8   (c) 12   (d) 15   (e) 16

22. Buses are dispatched at every 20 minutes from the depot. Assuming that there are no bus delays, on average how long do you have to wait at a bus stop if you go to the bus stop without prior information on when the buses left the depot?
   (a) 5   (b) 10   (c) 12.5   (d) 15   (e) 20

23. Amy’s video game lets her design the appearance of her character. There are 4 different body types and 3 faces to choose from. How many different appearances can Amy design for her character?
   (a) 3   (b) 4   (c) 7   (d) 12   (e) 20

24. Six pepperoni circles will fit exactly across the diameter of a 12-inch pizza when placed as shown. If a total of 24 pepperoni circles are placed on this pizza without overlap each other, what fraction of the pizza is covered by pepperoni circles?
   \[
   \text{Fraction covered by pepperoni circles} = \frac{\text{Number of pepperoni circles}}{\text{Total number of circles}}
   \]
   (a) \(\frac{1}{2}\)  (b) \(\frac{2}{3}\)  (c) \(\frac{3}{4}\)  (d) \(\frac{5}{6}\)  (e) \(\frac{7}{8}\)

25. The top of one tree is 16 feet higher than the top of another tree. The ratio of the heights of the two trees is 3:4. In feet, how tall is the taller tree?
   (a) 48  (b) 64  (c) 80  (d) 96  (e) 112

26. Of the 500 balls in a large bag, 80% are red and the rest are blue. How many of the red balls must be removed from the bag so that 75% of the remaining balls are red?
   (a) 25  (b) 50  (c) 75  (d) 100  (e) 150

27. The lengths of the sides of a triangle are three consecutive integers. The length of the shortest side is 30% of the perimeter. What is the length of the longest side?
   (a) 7  (b) 8  (c) 9  (d) 10  (e) 11
28. A square and a circle have the same area. What is the ratio of the side length of the square to the radius of the circle?
   (a) $\sqrt{\pi}$  (b) $\sqrt{\pi}$  (c) $\pi$  (d) $2\pi$  (e) $\pi^2$

29. A decorative window has a rectangle with semicircles on either end. The ratio of $AD$ to $AB$ is $3 : 2$, and $AB = 30$ inches. What is the ratio of the area of the rectangle to the sum of the areas of the two semicircles?

   \[ \frac{AD}{AB} = \frac{3}{2}, AB = 30 \text{ inches} \]

   \[ \frac{\text{Area of rectangle}}{\text{Sum of areas of two semicircles}} \]

   (a) $2 : 3$  (b) $3 : 2$  (c) $6 : \pi$  (d) $9 : \pi$  (e) $30 : \pi$

30. The two circles in the picture below have the same center $C$. The line $AD$ is tangent to the inner circle at $B$, $AC$ is 10, and the line $AD$ has length 16. What is the area between the two circles?

   \[ AC = 10, AD = 16 \]

   (a) $36\pi$  (b) $49\pi$  (c) $64\pi$  (d) $81\pi$  (e) $100\pi$

31. Two fifth of all the people in a room are wearing gloves, and three quarters of all the people in the room are wearing hats. What is the minimum number of people in the room wearing both gloves and hats?

   (a) 3  (b) 5  (c) 8  (d) 15  (e) 20

32. Amy bought a novel, and read twelve more than $\frac{1}{5}$ of the pages on the first day. On the second day, she read fifteen more than $\frac{1}{4}$ of the remaining pages. On the third day, she read eighteen more than $\frac{1}{3}$ of the remaining pages. She then realized that there were only 62 pages left in the novel. How many pages are in this book?

   (a) 120  (b) 180  (c) 240  (d) 300  (e) 360

33. The hundreds digit of a three-digit number is 2 more than the units digit. The digits of the three-digit number are reversed, and the result is subtracted from the original three-digit number. What is the units digit of the result?

   (a) 0  (b) 2  (c) 4  (d) 6  (e) 8
34. Semicircles $POQ$ and $ROS$ pass through the center $O$ of a circle. What is the ratio of the sum of the areas of the two semicircles to the area of the larger circle?

![Diagram](attachment:diagram.png)

(a) $\frac{\sqrt{2}}{4}$  
(b) $\frac{1}{2}$  
(c) $\frac{2}{\pi}$  
(d) $\frac{2}{3}$  
(e) $\frac{\sqrt{2}}{2}$

35. Given three numbers $10^8$, $5^{12}$, and $2^{24}$, which of the expressions below is the correct ordering of the numbers?

(a) $2^{24} < 10^8 < 5^{12}$  
(b) $2^{24} < 5^{12} < 10^8$  
(c) $5^{12} < 2^{24} < 10^8$  
(d) $10^8 < 5^{12} < 2^{24}$  
(e) $10^8 < 2^{24} < 5^{12}$

36. In the design shown below, the smallest black circle has a radius 2 inches, with each successive circle’s radius increasing by 2 inches. Approximately what percent of the design is black?

![Diagram](attachment:diagram.png)

(a) 42  
(b) 44  
(c) 45  
(d) 46  
(e) 48

37. In square $ABCE$, $AF = 2FE$ and $CD = 2DE$. What is the ratio of the area of triangle $BFD$ to the area of square $ABCE$?

![Diagram](attachment:diagram.png)

(a) $\frac{1}{6}$  
(b) $\frac{2}{9}$  
(c) $\frac{5}{18}$  
(d) $\frac{1}{3}$  
(e) $\frac{7}{20}$
38. Ten tiles have numbers 1 through 10 painted on them and are placed face down. You randomly pick a tile and throw a fare die, and multiply the two numbers from the tile and die. What is the probability that the product is a square number?
   (a) \( \frac{1}{10} \)  (b) \( \frac{1}{6} \)  (c) \( \frac{11}{60} \)  (d) \( \frac{1}{5} \)  (e) \( \frac{7}{30} \)

39. How many positive integer values of \( n \) can have both \( n/3 \) and \( 3n \) be three-digit whole numbers?
   (a) 12  (b) 21  (c) 27  (d) 33  (e) 34

40. Consider all rectangles with integer side lengths and a perimeter of 50 units. What is the difference between the largest and smallest areas of the rectangles?
   (a) 76  (b) 120  (c) 128  (d) 132  (e) 136

41. Three As, three Bs, and three Cs are placed in the nine spaces so that each row and column contain one of each letter. If A is placed in the upper left corner, how many such arrangements are possible?

![Diagram of a 3x3 grid with an A in the upper left corner]

   (a) 2  (b) 3  (c) 4  (d) 5  (e) 6

42. Given three boxes, there are three possible ways to choose a pair. If each pair weights either 122, 125 or 127 pounds, what is the sum of the weights of all three boxes?
   (a) 160  (b) 170  (c) 187  (d) 195  (e) 354

43. A ball is dropped from a height of 3 meters. On its first bounce, it rises to a height of 2 meters. It continues to fall and bounce back up to \( 2/3 \) of the height it reached on the previous bounce. On which bounce will it first not rise to a height at or above 0.5 meters?
   (a) 3  (b) 4  (c) 5  (d) 6  (e) 7

44. The average age of the 6 people in room A is 40. The average age of the 4 people in room B is 25. If the people in the two rooms gather together, what is the average age of the 10 people?
   (a) 32.5  (b) 33  (c) 33.5  (d) 34  (e) 35

45. An investment of $100 suffered a 15% loss in the first year. In the second year, however, the remaining investment showed a 20% gain. Over the two-year period, what is the change in the investment?
   (a) 5% loss  (b) 2% loss  (c) 1% gain  (d) 2% gain  (e) 5% gain
46. In how many ways can 10,001 be written as the sum of two primes?
(a) 0  (b) 1  (c) 2  (d) 3  (e) 4

47. On a dart board, the outer circle has radius 6 and the inner circle has radius 3. Three radii divide each circle into three congruent regions, with point values shown in the figure. The probability that a dart hits a given region is proportional to the area of the region. When two darts hit this board, the score is the sum of the point values in the two regions. What is the probability that the score is odd?

(a) \( \frac{17}{36} \)  (b) \( \frac{35}{72} \)  (c) \( \frac{1}{2} \)  (d) \( \frac{37}{72} \)  (e) \( \frac{19}{36} \)

48. Suppose you have five circles with radii 1, 2, 3, 4, and 5. Which of the graphs plots the point \((C, A)\), where \(C\) is the circumference and \(A\) is the area of the five circles.

(a)  \( \bullet \)  \( \bullet \)  \( \bullet \)  \( \bullet \)  \( \bullet \)  (b)  \( \bullet \)  \( \bullet \)  \( \bullet \)  \( \bullet \)  \( \bullet \)  (c)  \( \bullet \)  \( \bullet \)  \( \bullet \)  \( \bullet \)  \( \bullet \)  (d)  \( \bullet \)  \( \bullet \)  \( \bullet \)  \( \bullet \)  \( \bullet \)  (e)  \( \bullet \)  \( \bullet \)  \( \bullet \)  \( \bullet \)  \( \bullet \)

49. For any positive integer \(n\), define \(\langle n \rangle\) to be the sum of all positive factors of \(n\). For example, \(\langle 6 \rangle = 1 + 2 + 3 + 6 = 12\). Find \(\langle \langle 11 \rangle \rangle\).
(a) 13  (b) 20  (c) 24  (d) 28  (e) 30

50. A grid must have each of the digits 1 through 4 once in each row and once in each column. What number occupies the lower right-hand square in the grid?

(a) 1  (b) 2  (c) 3  (d) 4  (e) Cannot be determined
51. There are four consecutive integers such that the sum of the cubes of the first three numbers equals the cube of the fourth number. Find the sum of the four numbers.

52. A point \((x, y)\) is randomly chosen such that \(0 \leq x \leq 1\) and \(0 \leq y \leq 1\). What is the probability that \(y \leq 2x\)?

53. Find the smallest three-digit number whose remainders are 2, 3, 4 and 5 when it is divided by 3, 4, 5 and 6, respectively.

54. \[
\frac{3 \frac{8}{7}}{45} =
\]

55. If the pattern in the diagram continues, what fraction of the interior of the eighth triangle is shaded?

\[\text{1st} \quad \text{2nd} \quad \text{3rd} \quad \text{4th}\]
[1] (c)  [15] (d)  [29] (c)  [43] (c)
[2] (a)  [16] (e)  [30] (c)  [44] (d)
[3] (b)  [17] (c)  [31] (a)  [45] (d)
[4] (b)  [18] (a)  [32] (c)  [46] (a)
[5] (c)  [19] (e)  [33] (e)  [47] (b)
[6] (e)  [20] (c)  [34] (b)  [48] (a)
[7] (d)  [21] (d)  [35] (a)  [49] (d)
[8] (e)  [22] (b)  [36] (a)  [50] (b)
[9] (e)  [23] (d)  [37] (c)  [51] 18
[10] (c)  [24] (b)  [38] (c)  [52] $\frac{3}{4}$
[11] (b)  [25] (b)  [39] (a)  [53] 119
[12] (c)  [26] (d)  [40] (d)  [54] $\frac{25}{16}$
[13] (e)  [27] (e)  [41] (c)  [55] $\frac{7}{16}$
[14] (a)  [28] (b)  [42] (c)